Section - I

1.(b) Displacement $(\overrightarrow{\Delta S}) = 2r\sin\frac{\theta}{2} = 2r\sin\frac{180°}{2} = 2r$

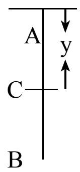
Distance $= r\theta = \pi r$

2.(c) $R_{max} = \frac{u^2}{g}$

or, $u^2 = R_{max} \times g$

Again $H = \frac{u^2}{2g} = \frac{80 \times 10}{2 \times 10} = 40$ m

3.(b)



Tension = wt. of part BC

$T = \frac{M}{L}(L - y)g = \frac{Mg(L - y)}{L}$

4.(a) $\Delta P = \frac{4T}{R} \propto \frac{1}{R}$

$\frac{\Delta P_1}{\Delta P_2} = \frac{R_2}{R_1} = \frac{1}{2}$

5.(c) Since cubical expensivity = 3 × linear expasivity

$= 3 \times \frac{\alpha}{3} = \alpha$

So remains stationary

6.(d) $mgh = ms\Delta\theta$

or, $\Delta\theta = \frac{gh}{s} = \frac{10 \times 21}{4200} = 0.05°C$

7.(c) $Q = \sigma AtT^4$

$\frac{Q_1}{Q_2} = \left(\frac{T_1}{T_2}\right)^4 = \frac{1}{16}$

8.(d) $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$

$\frac{9}{4} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$

or, $\frac{a_1 + a_2}{a_1 - a_2} = \frac{3}{2}$

or, $2a_1 + 2a_2 = 3a_1 - 3a_2$

or, $a_1 = 5a_2$

or, $\frac{a_1}{a_2} = \frac{5}{1}$

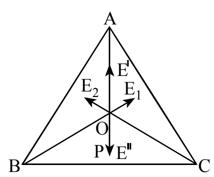
9.(b) Since $C' = \frac{\varepsilon_0 A}{d - t}$

Since $t = 0$

So, $C' = \frac{\varepsilon_0 A}{d}$

$C' = C_0$

10.(c)



Resultant of E_1 and E_2 = E' along OA of magnitude E_1.

The field due to charge at A = E" along OP of magnitude E_1

So resultant of E' and E" is zero.

11.(d) $P = \frac{V^2}{R} = \frac{V^2}{\rho l}A$ $\qquad P \propto \frac{1}{\rho}$

12.(a) $d \le \lambda$

13.(d) $\lambda = \frac{h}{\sqrt{2mK}}$ $\qquad \sqrt{2mK} = \frac{h}{\lambda}$

mK = cost. $\qquad k \propto \frac{1}{m}$

$m_e < m_p$ $\qquad \therefore\ K.E_e > K.E_p$

14.(c)

15.(b) $R = r_0 A^{1/3}$

$\frac{R_{Al}}{E_{sn}} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = 3 : 5$

16.(d) $f = \frac{v}{4l}$ $\qquad n = \frac{v}{4l} \Rightarrow l = \frac{v}{4n}$

17.(d) $v_n \propto \frac{1}{n}$ $\qquad r_n \propto n^2$

$f_n = \frac{mv^2}{r} \propto \frac{\left(\frac{1}{n}\right)^2}{n^2} \propto \frac{1}{n^4}$

18.(d)

19.(b)

20.(d) $I_2 + 10HNO_3 \rightarrow 2HIO_3 + 10NO_2 + 4H_2O$

21.(a)

22.(a) $\ddot{S} \rightarrow O$, with $S = O$ (double bond below S)

23.(c)

24.(b)

25.(b) Number of neutron = At. mass – At. no

70 – 30 = 40

26.(d)

27.(a) $0.1M\ Ca(OH)_2 = 0.2N\ Ca(OH)_2$

$N_1V_1 = N_2V_2$

$0.2 \times x = 0.1 \times 10$ $\qquad x = \frac{0.1 \times 10}{0.1} = 5\ ml$

28.(d) $W = ZIt$

$0.504 = \frac{1}{96500} \times x \times 2 \times 60 \times 60$

$x = 6.755$

$W = Z \times I \times t = \frac{8}{96500} \times 6.755 \times 2 \times 60 \times 60$

$= 4.032g \approx 4g$

29.(a) $K = \lim_{x \to 0} x\cos\frac{1}{x} = O \times$ finite value $= 0$

30.(c) $\frac{d}{dx}\log_e|x| = \frac{d}{d|x|}\log|x|\frac{d}{dx}|x| = \frac{1}{|x|}\cdot\frac{|x|}{x} = \frac{1}{x}$

31.(d) Let $y = 3^x$

$dy = 3^x \log_e 3 dx$ $\qquad \frac{1}{\log_e 3}dy = 3^x dx$

$I = \frac{1}{\log_e 3}\int\frac{dx}{1 + y^2} = \frac{1}{\log_e 3}\tan^{-1}(y) + c$

$= \frac{1}{\log_e 3}\tan^{-1}(3^x) + c$

32.(c) $dr = \Delta r = 5.1 - 5 = 0.1$
$A = \pi r^2$
$dA = 2\pi r\, dr = 2 \times \pi \times 5 \times 0.1 = \pi\ cm^2$

33.(b) Area $= \int_0^4 y\ \&\ x = \int_0^4 \frac{x^2}{4} dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4$
$= \frac{16}{3}$ sq. units

34.(c) $\frac{a + bw + w^2}{aw + bw^2 + c} = \frac{w^2(a + bw + cw^2)}{aw^3 + bw^4 + cw^2}$
$= \frac{w^2\,(a + bw + cw^2)}{a + bw + cw^2} = w^2$

35.(c) $x + \frac{1}{x} = 2x^2$
or, $3x^2 - x^2 - 1 = 0$
It is a cubic, so it has 3 roots. If one root is x = 1 then other 2 more roots.

36.(b) $S_\infty = \left(\frac{1}{7} + \frac{1}{7^3} + \ldots\ldots\right) + \left(\frac{2}{7^2} + \frac{2}{7^4} + \ldots\ldots\right)$
$= \frac{\frac{1}{7}}{1 - \frac{1}{7}} + \frac{\frac{2}{49}}{1 - \frac{1}{49}} = \frac{3}{16}$

37.(b) We have
A. adj(A) = |A| I
$\therefore\ \lambda = |A| = \begin{vmatrix} \cosh x & -\sinh x \\ -\sinh x & \cosh x \end{vmatrix}$
$= \cosh^2 x - \sinh^2 n = 1$

38.(d) Set A contains n elements number of elements in A × A = n^2 no. of relations on A = number of subsets of A × A = 2^{n^2}

39.(d) We have
$c_0 + c_1x + c_2x^2 + \ldots.. + c_nx^n\ (1 + x)^n$
Putting x = 3
$c_0 + 3c_1 + 9c_2 + \ldots\ldots.. + 3^nc_n = 4^n$

40.(d) Number selection $= {}^5C_3 \times {}^5C_2$
$= 10 \times 10 = 100$

41.(a) $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{-4 - 6 - 6}{\sqrt{4 + 4 + 1}} = -\frac{16}{\sqrt{9}} = -\frac{16}{3}$

42.(b) Here a, b, c are in A.P.
$\Rightarrow b = \frac{a + c}{2}$
$\Rightarrow a - 2b + c = 0$
$\therefore$ (1 – 2) lines on the line ax + by + c = 0
Hence, ax + by + c = 0 represents a family of lines passing trough (1, –2)

43.(b) Here $h^2 - ab = 6^2 - 4.9 = 0$
$\therefore$ lines are real and coincident.

44.(d) $(x - 5)^2 + (y - 7)^2 = 3^2(\cos^2\theta + \sin^2\theta) = 9$
$\therefore$ It is a circle

45.(d) $c = \frac{a}{m} = \frac{4}{2} = 2$

46.(a) $l = m = n$
$\therefore\ l^2 + m^2 + n^2 = 1$
$3l^2 = 1 \qquad \therefore\ l = m = n = \pm\frac{1}{\sqrt{3}}$

47.(b) $\frac{b\cos C + b\cos A + \cos B + a\cos B}{b(c + a)}$
$= \frac{c + a}{b(c + a)} = \frac{1}{b}$

48.(b) Here comparing with asinx + bcosx = c
a = 1, b = 1, c = 2
$\therefore\ c > \sqrt{a^2 + b^2} = \sqrt{2}$
$\therefore$ There is no solution.

49.a	50.a	51.b	52.c	53.d	54.a
55.b	56.b	57.a	58.c	59.b	60.d

Section – II

61.(d) $g = \frac{GM}{R^2}$
$\frac{g_{planet}}{g_{earth}} = \frac{M_p}{M_e} \times \left(\frac{R_e}{R_p}\right)^2 = \frac{1}{2} \times (2)^2 = 2$
$\therefore\ g_p = 2g$

62.(b) $F_{max} = m\omega^2A = m\left(\frac{2\pi}{T}\right)^2 A$
$= \frac{4\pi^2}{T^2}\, mA$
$= \frac{4\pi^2 \times 50 \times 10^{-3} \times 0.1}{0.1^2} = 20N$

63.(b) Total weight = wt. of water displaced
$m_1g + m_2g = (v_1 + v_2)\,\rho_w g$
$m_1 + 10 = \left(\frac{m_1}{11} + \frac{10}{0.2}\right) \times 1$
$m_1 = 44$ gm

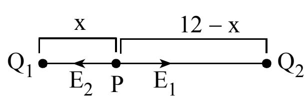
64.(c) Tension (= 150) < weight
So acceleration down ward
mg – T = ma
or, $20 \times 9.8 - 150 = 20a$
or, $a = 2.3\ m/s^2$

65.(a) $\frac{\Delta l}{l_1\alpha} = \Delta\theta$
$\frac{0.0005\ mm}{1\ mm \times 1.32 \times 10^{-5}} = \Delta\theta$
$\Rightarrow\ \Delta\theta = 37.8°C$

66.(a) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
$T_2 = 395.85$ K
$\therefore$ Rise in temperature $\Delta T = T_2 - T_1$
$= 395.85 - 300$
$= 95.85$ K

67.(c) $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$
or, $f \propto \frac{1}{l}$ or, $\frac{f_2}{f_1} = \frac{l_1}{l_2}$
or, $\frac{320}{256} = \frac{l}{l - 10}$ $\Rightarrow l = 50$ cm

68.(c)



$E_1 = E_2$

or, $\dfrac{9\times10^9 Q_1}{x^2} = \dfrac{9\times10^9 Q_2}{(12-x)^2}$

or, $\dfrac{2\times10^{-6}}{x^2} = \dfrac{8\times10^{-6}}{(12-x)^2}$ or, $4x^2 = (12-x)^2$

or, $2x = 12 - x$ or, $x = 4$ cm

69.(b) $d\sin\theta = n\lambda$

or, $\dfrac{1}{N}\sin 30° = 3 \times 5555 \times 10^{-10}$

or, $N = 3000$ lines/cm

70.(a) $E = I_1(R + r)$

or, $E = 3(1.8 + r)$

Again $E = 2\,(2.9 + r)$ $\Rightarrow r = 0.4\ \Omega$

71.(c) $E = \dfrac{1}{2}B\omega l^2$

$= \dfrac{1}{2}\times 0.4 \times 32 \times 0.5^2$

$= 1.6$ V

72.(c) $r = \dfrac{\sqrt{2mK}}{eB} = \sqrt{\dfrac{2\times9.1\times10^{-31}\times150\times1.6\times10^{-19}}{1.6\times10^{-19}\times0.1}}$

$= 4\times10^{-4}$ m

73.(c) $\dfrac{1}{\lambda_L} = R\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)$

$\dfrac{1}{\lambda_B} = R\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right)$

$\dfrac{\lambda_B}{\lambda_L} = \dfrac{27}{5}$

$\therefore\ \lambda_B = \dfrac{27}{5}\times 1215\ Å = 6561\ Å$

74.(b) $\dfrac{C}{C_0} = \left(\dfrac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

or, $\dfrac{1\times10^6}{4\times10^6} = \left(\dfrac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

or, $T_{1/2} = 10$ hr.

Again $\dfrac{C}{C_0} = \left(\dfrac{1}{2}\right)^{\frac{100}{10}}$

$C = 4\times10^6 \times \left(\dfrac{1}{2}\right)^{10} = 3.9\times10^3$/sec.

75.(c)

$$\begin{matrix}CH_2COOK\\ |\\ CH_2COOK\end{matrix} + 2H_2O \xrightarrow{\text{Electrolysis}} \underbrace{\begin{matrix}CH_2\\ \|\\ CH_2\end{matrix} + 2CO_2}_{\text{At anode}} + \underbrace{2KOH + H_2}_{\text{At cathode}}$$

76.(c)

$$(CH_3)_2C = CHCH_3 \xrightarrow{NaI_4/KMnO_4} (CH_3)_2C = O + [CH_3CHO] \rightarrow CH_3COOH$$

77.(a)

$$CaC_2 + N_2 \xrightarrow{800°C} \underbrace{CaCN_2 + C}_{\text{Nitrolim}}$$

78.(c) $\dfrac{\text{Wt. of metal oxide}}{\text{EW of metal oxide}} = \dfrac{\text{Wt. of water}}{\text{EW of oxygen}}$

$\dfrac{0.426}{x+8} = \dfrac{0.12}{9}$ $x = 23.95 \approx 24$

79.(c) EW of element $= \dfrac{\text{Mass of element}}{\text{Mass of oxygen}} \times 8$

$= \dfrac{53}{47}\times 8 = 9.0212 \approx 9$

At. wt. = Eq. wt. × valency

$= 9 \times 3 = 27$

80.(d) $SrF_2 \rightleftharpoons Sr^{++} + 2F^-$

$\qquad\qquad S \qquad 2S$

$NaF \rightarrow Na^+ + F^-$

$\qquad\quad 0.1 \qquad 0.1$

$Sr^{++} = S,\ F^- = (2S + 0.1)$

$K_{sp} = S \times (2S + 0.1)^2$

$8\times10^{-10} = S \times (0.1)^2$ $[2S + 0.1 \approx 0.1]$

$8\times10^{-10} = S \times 10^{-2}$

$S = 8\times10^{-8}$

81.(a) pH = 10.65

pOH = 3.35

pOH = $-\log[OH^-]$

$3.35 = -\log[OH^-]$

$OH^- = 4.47\times10^{-4}$ moles

$\therefore$ $[OH^-]$ in 250 ml

$[OH^-] = \dfrac{4.47\times10^{-4}\times250}{1000} = 1.12\times10^{-4}$ moles/L

No. of moles of $Ca(OH)_2$ dissolved

$= \dfrac{1.12\times10^{-4}}{2} = 0.56\times10^{-4}$

82.(b) Here $y = \cos^{-1}\dfrac{1-(\log x)^2}{1+(\log x)^2}$

$= 2\tan^{-1}(\log x)$

$\therefore\ \dfrac{dy}{dx} = 2\dfrac{d}{dx}\tan^{-1}(\log x)$

$= 2\dfrac{d}{d(\log x)}\tan^{-1}(\log x)\dfrac{d}{dx}\log x$

$= 2\dfrac{1}{1+(\log x)^2}\times\dfrac{1}{x}$

At h = e, $\dfrac{dy}{dx} = \dfrac{2}{1+1^2}\cdot\dfrac{1}{e} = \dfrac{1}{e}$

83.(b) $\int \sin^{-1}x\,dx$

$= x\sin^{-1}x - \int\left(\dfrac{d}{dx}\sin^{-1}x\int dx\right)dx$

$= x\sin^{-1}x - \int\dfrac{x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1}x + \dfrac{1}{2}\int\dfrac{-2x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1}x + \dfrac{1}{2}\cdot 2\sqrt{1-x^2} + c$

$= x\sin^{-1}x + \sqrt{1-x^2} + c$

84.(d) $f(x) = \cos x - \cos^2 x + \cos^3 x - \ldots\ldots\ldots + \infty$

$= \dfrac{\cos x}{1+\cos x}$

$\int f(x)\,dx = \int \frac{\cos x\,dx}{1+\cos x}$

$\int \frac{1+\cos x-1}{1+\cos x}dx$

$= \int dx - \int \frac{1}{1+\cos x}dx$

$= x - \frac{1}{2}\int \frac{dx}{\cos^2\frac{x}{2}}$

$= x - \frac{1}{2}\int \sec^2\frac{x}{2}\,dx = x - \tan\frac{x}{2} + c$

85.(a) At certain time t,
A = area, r = radius, p = perimeter
$A = \pi r^2,\ p = 2\pi r$

$\frac{dA}{dt} = K \qquad 2\pi r\frac{dr}{dt} = K$

$\frac{dr}{dt} = \frac{K}{2\pi r} \qquad \Rightarrow \frac{dr}{dt} \propto \frac{1}{r}$

86.(a) Let $\sqrt{5+12i} = x + iy$ then

$x^2 = \frac{\sqrt{a^2+b^2}+a}{2}, \qquad y^2 = \frac{\sqrt{a^2+b^2}-a}{2}$

$= \frac{\sqrt{5^2+12^2}+5}{2} \qquad = \frac{\sqrt{5^2+12^2}-5}{2}$

$= 9 \qquad = 4$

$x = \pm 3 \qquad y = \pm 2$

Since b = 12 > 0, so square roots are $\pm(3+2i)$

87.(c) a, b, c are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P.

$\Rightarrow 1+\frac{b+c}{a}, 1+\frac{a+c}{b}, 1+\frac{a+b}{c}$ are in A.P.

$\Rightarrow \frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

$\Rightarrow \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

88.(a) $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix}$

$= \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$

as $\alpha+\beta+\gamma = 0$

89.(c) $t_n = \frac{(2n-1)}{n!} = \frac{2}{(n-2)!} + \frac{1}{(n-1)!}$

$S_\infty = 2\sum\frac{1}{(n-2)!} + \sum\frac{1}{(n-1)!}$

$= 2e + e = 3e$

90.(c) Total $= \frac{6!}{2!} = 360$

O's come together = 5! = 120
Required = 360 – 120 = 240

91.(c) $m + 3m = -\frac{2h}{b^2}$

$m = -\frac{h}{2b^2}$

$m \times 3m = \frac{a^2}{b^2}$

$3m^2 = \frac{a^2}{b^2}$

$3\frac{b^2}{4b^4} = \frac{a^2}{b^2}$

$\therefore\ h = \frac{2}{\sqrt{3}}\ ab$

92.(b) $3x^2 - 3y^2 = 25 \qquad$ or, $x^2 - y^2 = \frac{25}{3}$ ….. (1)

Conjugate hyperbola f (1) is

$x^2 - y^2 = -\frac{25}{3}$ ……….. (2)

As (1) and (2) are rectangular hyperbola

$\therefore\ e_1 = e_2 = \sqrt{2}$

$\therefore\ e_1^2 + e_2^2 = 4$

93.(a) Equation of parallel plane
$x - 2y + 2z = K$

Also $\frac{|1-4+6-K|}{\sqrt{1+4+4}} = 1$

or, $|3 - K| = 3$
or, $3 - K = \pm 3$
$\therefore\ K = 0, K = 6$
Required plane $x - 2y + 2z = 6$

94.(c) $\tan^{-1}x + \tan^{-1}y = \pi - \tan^{-1}z$

or, $\tan^{-1}\frac{x+y}{1-xy} = \pi - \tan^{-1}z$

$\frac{x+y}{1-xy} = \tan^{-1}(\pi - \tan^{-1}z)$

$x + y = -z(1 - xy)$

or, $x + y + z = xyz$

$\frac{1}{yz} + \frac{1}{zx} + \frac{1}{xy} = 1$

95.(d) $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{a}{2R}}{\frac{b^2+c^2-a^2}{2bc}} = \frac{\frac{abc}{R}}{b^2+c^2-a^2}$

$= \frac{4\Delta}{b^2+c^2-a^2}$

96.(c) Here coplanar $\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

Expanding abc + 2 = a + b + c

97.d 98.a 99.b 100.b

…The End…