Section - I

1. (b) $\vec{A}.\vec{B}=0$

or, $(\cos\omega t\hat{i}+\sin\omega t\hat{j}).\left(\cos\frac{\omega t}{2}\hat{i}+\sin\frac{\omega t}{2}\hat{j}\right)=0$

or, $\cos\omega t\cos\frac{\omega t}{2}+\sin\omega t.\sin\frac{\omega t}{2}=0$

or, $\cos\left(\omega t-\frac{\omega t}{2}\right)=0=\cos\frac{\pi}{2}$

or, $\omega t-\frac{\omega t}{2}=\frac{\pi}{2}$

or, $\frac{\omega t}{2}=\frac{\pi}{2}$

or, $t=\frac{\pi}{\omega}$

2. (b) If a body have constant velocity then its speed must be constant.

3. (d) $R-mg=ma$

or, $R=m(g+a)$

4. (c) Moment of inertia first decreases
So the angular speed increases the again the moment of inertia increases so the angular speed decreases.

5. (b) Metal of higher linear expansivity lies on convex side.

6. (a) Black body is good absorbed as well as good radiator.

7. (a) $n=\frac{v}{4l}$

if length is double then

$n'=\frac{v}{4l'}=\frac{v}{4\times 2l}=\frac{n}{2}$

8. (d) $I_{max}=I+I+2\sqrt{II}\cos 0°$
$=4I$

9. (a) $F=EQ=\frac{\sigma}{2\varepsilon_0}$

$=\frac{Q^2}{2\varepsilon_0 A}$

$=\frac{(CV)^2}{2C.d}=\frac{1}{2}\frac{CV^2}{d}$

10. (d) When charge θ is brought close to metal cube then equal and opposite charge is induced near to charge and equal and same charge is induced away from it keep.
Total charge zero inside cube.

11. (c) $\frac{R'}{R}=\left(\frac{2l}{l}\right)^2=4$

$\therefore\ R'=4R$

12. (d) $I_1^2R_1t=I_2^2R_2t$

or, $\left(\frac{E}{R_1+r}\right)^2R_1=\left(\frac{E}{R_2+r}\right)^2.R_2$

or, $\left(\frac{R_2+r}{R_1+r}\right)^2=\frac{R_2}{R_1}$

or, $\frac{R_2+r}{R_1+r}=\sqrt{\frac{R_2}{R_1}}$

or, $R_2\sqrt{R_1}+r\sqrt{R_1}=R_1\sqrt{R_2}\sqrt{R_2}+r\sqrt{R_2}$

or, $R_2\sqrt{R_1}-R_1\sqrt{R_2}=r\sqrt{R_2}-r\sqrt{R_1}$

or, $\sqrt{R_1}\sqrt{R_2}(\sqrt{R_2}-\sqrt{R_1})=r(\sqrt{R_2}-\sqrt{R_1})$

or, $r=\sqrt{R_1}\sqrt{R_2}$

13. (c) $I_{max}=\sqrt{\left(\frac{I_1}{\sqrt{2}}\right)^2+\left(\frac{I_2}{\sqrt{2}}\right)^2}$

$=\sqrt{\frac{I_1^2+I_2^2}{2}}$

14. (c) Convex mirror form virtual image for real object

15. (c)

16. (d) $\frac{\left(\frac{e}{m}\right)_p}{\left(\frac{2e}{4m}\right)_\alpha}=\frac{e}{m}\times\frac{4m}{2e}=2:1$

17. (d) $\beta=60$

Voltage gain $=\frac{V_{out}}{V_{in}}=\frac{I_cR_{out}}{I_bR_{in}}$

$=\beta\times\frac{5000}{500}$

$=600$

18. (c)

19. (a)

20. (a) In basic medium

$2K\overset{+7}{Mn}O_4+2KOH\longrightarrow 2K_2\overset{+6}{Mn}O_4+H_2O+[O]$

Change in O.N. of Mn = +7 – (+6) = 1

EW of $KMnO_4=\frac{158}{1}=158$

$\frac{W}{E}=\frac{N\times V_{ml}}{1000}$

$\frac{x}{158}=\frac{0.1\times 100}{1000}$

$x=\frac{0.1\times 100\times 158}{1000}=1.58$

21. (b) $1.8L=1.8\times 1000=1800$ ml

Mass = V × D = 1800 × 1 = 1800g

18g water = 1 mole

1800g water $=\frac{1}{18}\times 1800=100$ moles

22. (a)

23. (a) (a) 24 mg
(b) 1 mole = 30g
0.9 moles = 30 × 0.9 = 27g
(c) 22.4L N_2 = 28g
(d) 6.023×10^{23} O_2 molecules = 32g

24. (a)

$N_2H_4\longrightarrow Y+10e^-$

$\longrightarrow$ (7 + 3) moles ⇏ NH_3^-

$\longrightarrow$ (7 + 1) moles = $\overset{-1}{N}\overset{+1}{H}$

25. (b)
26. (d)
27. (c)
28. (b)
29. (c) No. of non-empty proper subsets $= 2^n - 2$
$= 2^4 - 2 = 14$

30. (c) $|3 - x| < 4$
$\Rightarrow -4 < 3 - x < 4$
$\Rightarrow -7 < -x < 1$
$\Rightarrow -1 < x < 7$

31. (a) $\dfrac{b - c\cos A}{a - c\cos B} = \dfrac{a\cos C + c\cos A - c\cos A}{b\cos C + c\cos B - c\cos B} = \dfrac{a}{b}$

32. (b) $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = \sin^{-1}\left[\sin\left(-\dfrac{\pi}{3}\right)\right] = -\dfrac{\pi}{3}$

33. (d) Here, n = 10, r = 2
Total no. of hand shakes $= {}^nC_r = {}^{10}C_2 = 45$

34. (a) The system has no solution if D = 0
i.e. $\begin{vmatrix} k & 3 \\ 1 & 2 \end{vmatrix} = 0$
$\Rightarrow 2k - 3 = 0 \Rightarrow k = \dfrac{3}{2}$

35. (b) $\log_e(1 - 2x)$ is valid if $-1 \le 2x < 1$
$\Rightarrow -\dfrac{1}{2} \le x < \dfrac{1}{2}$

36. (b) We have, $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n$ … (i)
Put x = 3 in (i), we get
${}^nC_0 + 3.{}^nC_1 + 3^2.{}^nC_2 + \dots + 3^n.{}^nC_n = 4^n$

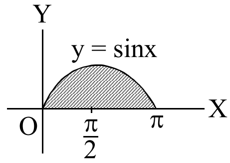
37. (c) $k.2^{k-1} = 80$
$\Rightarrow k = 5$

38. (d) $y = e^{-x}$
$\dfrac{dy}{dx} = -e^{-x}$
$\dfrac{d^2y}{dx^2} = e^{-x}$
$\dfrac{d^3y}{dx^3} = -e^{-x} = -y$

39. (c) $\displaystyle\int \frac{dx}{1 - x^2} = \frac{1}{2}\ln\left|\frac{1 + x}{1 - x}\right| + c$

40. (d) Slope of normal $= -\dfrac{dx}{dy}$
$\Rightarrow \tan\dfrac{3\pi}{4} = -\left(\dfrac{dx}{dy}\right)_{(3,\,4)}$
$\Rightarrow \left(\dfrac{dy}{dx}\right)_{(3,\,4)} = 1$
$\therefore f'(3) = 1$

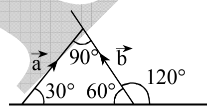
41. (b) Area $= 2\displaystyle\int_0^{\pi/2} \sin x\, dx$
$= 2\,[-\cos x]_0^{\pi/2} = 2$ sq. units



42. (b) $\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\sinh x = \dfrac{e^x - e^{-x}}{2}$
Then, $\cosh x + \sinh x = \dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2} = e^x$

43. (b) Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$
$= \dfrac{(2)\,(1) + (3)\,(2) + (-2)\,(3)}{\sqrt{1^2 + 2^2 + 3^2}}$
$= \dfrac{2}{\sqrt{14}}$

44. (c) $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$
$= 1 + 1 + 0 = 2$
$\therefore |\vec{a} + \vec{b}| = \sqrt{2}$



45. (a) Ratio $= -\dfrac{x_1}{x_2} = \dfrac{2}{3} = 2 : 3$

46. (d) $(y + 1)^2 = -(x - 1)$
$\therefore$ Vertex = (1, –1) which lies in fourth quadrant.

47. (a) $m_1 + m_2 = m_1m_2$
or, $\dfrac{2h}{7} = \dfrac{4}{-7}$ $\Rightarrow h = -2$

48. (d) $a^2 = 25 \Rightarrow a = 5$
$b^2 = 144 \Rightarrow b = 12$
Length of latus rectum $= \dfrac{2b^2}{a} = \dfrac{2 \times 144}{5} = \dfrac{288}{5}$

49.a	50.b	51.b	52.c	53.b	54.a
55.d	56.a	57.c	58.a	59.b	60.c

Section – II

61. (b)
← d → Catch
Police Culprit
$S_p - S_c = d$
or, $vt - \dfrac{1}{2}at^2 = d$
or, $at^2 - 2vt + 2d = 0$
$\therefore t = \dfrac{2v \pm \sqrt{4v^2 - 4a \times 2d}}{2a}$
Time must be real so $4v^2 - 8ad \ge 0$

62. (b) $\dfrac{mv^2}{r} = \mu mg$
or, $v = \sqrt{\mu rg} = \sqrt{0.6 \times 150 \times 10}$
$= 30$ m/s

63. (d) $\Delta PE = \left\{-\frac{GMm}{r} - \left(-\frac{GMm}{R}\right)\right\}$

$= GMm\left[\frac{1}{R} - \frac{1}{R+nR}\right]$

$= gR^2.m\left(\frac{n+1-1}{R(n+1)}\right)$

$= mgR\left(\frac{n}{n+1}\right)$

64. (a) Heat lost by steam = Heat gained by water

or, $m \times 540 + m(100-90) = 22(90-20)$

or, $m = \frac{22 \times 70}{560} = 2.75g$

$\therefore$ Water $= 22 + 2.75 = 24.8g$

65. (c) $P_T = P_i^{\circ} + P_i^N$

$= (n_0 + n_N)\frac{RT}{V}$

$= \left(\frac{P_1V_1}{RT} + \frac{P_2V_2}{RT}\right)\frac{RT}{V}$

$= \frac{1 \times 1 + 2 \times 0.5}{1} = 2$ atm

66. (b)

$200 \nearrow 205$ Hz, $200 \searrow 195$ Hz

Again 10 beats/s with 420 Hz produced so

$420 \nearrow 430$ Hz, $420 \searrow 410$ Hz

Here second harmonics of 205 Hz can be 410 Hz so frequency is 205 Hz.

67. (d) When capacitors are placed in parallel then

$C_p = 2C$

$V = V_0 e^{-\frac{t}{GR}}$

or, $\frac{6}{2} = 6\,e^{-\frac{t}{2CR}}$

or, $\frac{1}{2} = \frac{1}{e^{\frac{t}{2CR}}}$

or, $e^{\frac{t}{2CR}} = 2$

or, $\frac{t}{2CR} = ln2$

or, $t = ln2 \times 2CR$ ..... (i)

When placed in series

$C_s = \frac{C}{2}$

or, $\frac{6}{2} = 6e^{-\frac{t'}{2C_sR}}$

or, $\frac{1}{2} = \frac{1}{e^{\frac{t'}{2C_sR}}}$

or, $e^{\frac{t'}{2C_sR}} = 2$

or, $\frac{t'}{\frac{C}{2}R} = ln2$

or, $t' = ln2 \times \frac{CR}{2}$ ..... (ii)

Dividing (ii) by (i)

$\frac{t'}{10} = 2 \times 2$

or, $t' = 2.5$ s

68. (b) $I_1^2R_1t_1 = I_2^2R_2t_2 = ms\Delta\theta$

or, $\left(\frac{V}{R_1}\right)^2 \times R_1 \times 15 = \left(\frac{V}{R_2}\right)^2 R_2t_2$

or, $\frac{V^2}{R_1} \times 15 = \frac{V^2}{R_2} \times t_2$

or, $\frac{15}{R} = \frac{t_2}{\frac{2}{3}R}$

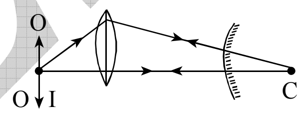
or, $t_2 = 15 \times \frac{2}{3} = 10$ min

69. (b) $I = \frac{d\phi}{R} = \frac{\frac{NAdB}{dt}}{R}$

$= \frac{10 \times 10 \times 10^{-4} \times 10^8 \times 10^{-4}}{20}$

$= 5A$

70. (c)



For lens,

$V = \frac{fu}{u-f} = \frac{20 \times 30}{30-20} = \frac{600}{10} = 60$ cm

$d = v - r = 60 - 10 = 50$ cm

71. (a) $(\mu - 1)\,t = n\lambda$

or, $\lambda = \frac{(1.5-1) \times 6 \times 10^{-6}}{5}$

$= 6 \times 10^{-7}$ m $= 6000$ Å

72. (b) Power $= \frac{nhc}{t\lambda}$

or, Power $= \frac{n}{t}\frac{hc}{h} \times P$

or, $\frac{n}{t}P = \frac{\text{Power}}{C}$

Force (F) = Rate of change in momentum

$= 1.6\frac{n}{t}P$

$= 1.6 \times \frac{\text{Power}}{C}$

$= 1.6 \times \frac{60}{3 \times 10^8}$

$= 3.2 \times 10^{-7}$ N

73. (b) $E = \phi + eV_s$
$= 2.75 + 10$
$= 12.75$ eV
Now, $E - E_1 = 12.75$
or, $E = 12.75 - 13.6$
or, $E = -0.85$ eV
$\therefore\ -\dfrac{13.6}{n^2} = -0.85$ or, $n = 4$

74. (b) $\dfrac{C}{C_0} = \left(\dfrac{1}{2}\right)^{\frac{t}{T_{1/2}}}$
or, $\dfrac{3000}{6000} = \left(\dfrac{1}{2}\right)^{\frac{140}{T_{1/2}}}$
or, $T_{1/2} = 140$ days
Again, $\dfrac{C}{C_0} = \left(\dfrac{1}{2}\right)^{\frac{t}{T_{1/2}}}$
or, $\dfrac{6000}{C_0} = \left(\dfrac{1}{2}\right)^{\frac{280}{140}}$
or, $C_0 = 24000$ dps

75. (d)
$$HC \equiv CH \xrightarrow[\text{(ii) } H_2O/Zn]{\text{(i) } O_3} \underset{\text{Glyoxal}}{H-\underset{\underset{O}{\|}}{C}-\underset{\underset{O}{\|}}{C}-H}$$

76. (b) $NH_4Cl + KNO_2 \xrightarrow{\Delta} KCl + NH_4NO_2$
$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

77. (b)

78. (d) $\dfrac{W}{E} = \dfrac{N \times V_{ml}}{1000}, \dfrac{0.23}{E} = \dfrac{0.1 \times 30}{1000}$
$E = \dfrac{0.23 \times 1000}{0.1 \times 30} = \dfrac{230}{3} = 76.67$
$MW = 2 \times 76.67 = 153.3$
Metal oxide = 153.3 $\quad M = 153.3 - 16$
$M + 0 = 153.3 \quad M = 137.3$

79. (d) $Mg(OH)_2 \rightleftharpoons \underset{S}{Mg^{++}} + \underset{2S}{2OH^-}$ (S)
$K_{sp} = 4s^3$
$S = \sqrt[3]{\dfrac{1 \times 10^{-11}}{4}} = 1.358 \times 10^{-4}$ mol L^{-1}
$OH^- = 2S = 2 \times 1.358 \times 10^{-4} = 2.716 \times 10^{-4}$ mol/L^{-1}
pOH = log[2.716×10^{-4}] = 3.566
pH = 14 – pOH = 14 – 3.566 = 10.43

80. (c) Number of moles of O_2 used up = 25% = $\dfrac{25}{100}$
= 0.25 mole
$$\underset{'g'}{2SO_2} + \underset{'g'}{O_2} \rightleftharpoons \underset{'g'}{2SO_3}$$
Initial moles: 4 mol 4 mole
At equation (4–2 moles) (4–4 × 0.25 moles) 2 mole
Total no. of moles = 2 + 3 + 2 = 7 moles.

81. (a) 100g of $CaCO_3 = 6.023 \times 10^{23}$ no. of C-atoms
25g of $CaCO_3 = \dfrac{6.023 \times 10^{23}}{100} \times 25 = 1.5 \times 10^{23}$

82. (d) For ${}^{7-x}P_{x-3}$ to be defined, $7 - x > 0 \Rightarrow x < 7$
And, $x - 3 \ge 0 \Rightarrow x \ge 3$; $7 - x \ge x - 3$
$\Rightarrow x \le 5$
$\therefore\ x \in \{3, 4, 5\} \Rightarrow f(3) = 1, f(4) = 3, f(5) = 2$
Range = {1, 2, 3}

83. (d) $\cos\theta = \sin 105° + \cos(90 + 15)$
or, $\cos\theta = \sin 105 - \sin 15$
$= 2\cos\left(\dfrac{105 + 15}{2}\right)\sin\left(\dfrac{105 - 15}{2}\right)$
$= 2\cos 60.\sin 45 = \dfrac{1}{\sqrt{2}} = \cos\dfrac{\pi}{4}$
$\therefore\ \theta = 2n\pi \pm \dfrac{\pi}{4}$

84. (c) $\vec{d}_1 = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{d}_2 = -3\vec{i} - 2\vec{j} + \vec{k}$
$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix} = 8\vec{i} - 10\vec{j} + 4\vec{k}$
Area $= \dfrac{1}{2}|\vec{d}_1 \times \vec{d}_2| = \dfrac{1}{2}\sqrt{8^2 + (-10)^2 + 4^2} = 3\sqrt{5}$

85. (b) $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = -2$
$A^{-1} = \dfrac{1}{|A|}$. Adj. A
(2, 3) element of $A^{-1} = \dfrac{1}{|A|} \times$ cofactor of (3, 2) element of A
$= -\dfrac{1}{2}\left(-\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix}\right) = -\dfrac{1}{2} \times 2 = -1$

86. (b) $\left[\left(\dfrac{1-i}{\sqrt{2}}\right)^2\right]^4 + \left[\left(\dfrac{1+i}{\sqrt{2}}\right)^2\right]^4$
$= \left(\dfrac{1 - 2i - 1}{2}\right)^4 + \left(\dfrac{1 + 2i - 1}{2}\right)^4 = (-i)^4 + i^4$
$= 1 + 1 = 2$

87. (c) $\dfrac{1}{4}\left(\dfrac{4}{3 \times 7} + \dfrac{4}{7 \times 11} + \dfrac{4}{11 \times 15} + \ldots\right)$
$= \dfrac{1}{4}\left(\dfrac{1}{3} - \dfrac{1}{7} + \dfrac{1}{7} - \dfrac{1}{11} + \dfrac{1}{11} - \dfrac{1}{15} + \ldots\right)$
$= \dfrac{1}{4} \times \dfrac{1}{3} = \dfrac{1}{12}$

88. (d) $x^2 - 7|x| + 12 = 0$
or, $|x|^2 - 4|x| - 3|x| + 12 = 0$
or, $|x|(|x| - 4) - 3(|x| - 4) = 0$
or, $(|x| - 4)(|x| - 3) = 0$
i.e. $|x| = 4 \Rightarrow x = -4, 4$
& $|x| = 3 \Rightarrow x = -3, 3$
$\therefore\ x = -4, -3, 3, 4$
No. of real solutions = 4

89. (a) $a = 1, b = -2, c = 2$

D.C's $l = \frac{1}{\sqrt{6}}, m = -\frac{1}{\sqrt{6}}, n = \frac{2}{\sqrt{6}}$

Required projection = $(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$

$= 2 \times \frac{1}{\sqrt{6}} + (-6)\left(-\frac{1}{\sqrt{6}}\right) + 2 \times \frac{1}{\sqrt{6}}$

$= 2\sqrt{6}$

90. (a) Distance between centres = 5

When two circles intersect each other then difference between their radii < distance between centre

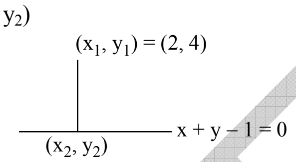
$\Rightarrow \quad r - 3 < 5 \Rightarrow r < 8$ ….. (i)

Sum of their radii > distance between centres.

$\Rightarrow \quad r + 3 > 5 \quad \Rightarrow r > 2$ …… (ii)

From (i) and (ii), $2 < r < 8$

91. (a) Let the coordinates of foot of perpendicular be (x_2, y_2)



Then, $\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$

i.e. $\frac{x_2 - 2}{1} = \frac{y_2 - 4}{1} = -\frac{(2 + 4 - 1)}{1 + 1}$

From 1st & 3rd, $x_2 = -\frac{1}{2}$

From 2nd & 3rd, $y_2 = \frac{3}{2}$

Required point = $\left(-\frac{1}{2}, \frac{3}{2}\right)$

92. (b) Let $P = \lim_{x \to 0} x^x$

Then, $lnP = \lim_{x \to 0} x\, lnx$

$= \lim_{x \to 0} \frac{lnx}{\frac{1}{x}}$

$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$ [By L' Hospital rule]

$= 0$

$\therefore \quad P = e^\circ = 1$

93. (a) $y = x^2 + \frac{1}{y}$

$\Rightarrow \quad y^2 = x^2y + 1$

Diff. both sides w.r. to x

$2y\frac{dy}{dx} = x^2\frac{dy}{dx} + y.2x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2xy}{2y - x^2}$

94. (b) Let $f(x) = \left(\frac{1}{x}\right)^x = x^{-x} = e^{-xlnx}$

$f'(x) = e^{-xlnx}(-1 - lnx)$

For max. or min, $f'(x) = 0 \Rightarrow x = \frac{1}{e}$

$f''(x) = e^{-xlnx}\left(-\frac{1}{x}\right) + (-1 - lnx)^2 e^{-xlnx}$

$\therefore \quad f''\left(\frac{1}{e}\right) = -e.e^{1/e} < 0$ (Max. value)

Maximum value = $e^{1/e}$

95. (c) $\int (\sin^{-1}x + \cos^{-1}x)\,dx$

$= \int \frac{\pi}{2}\,dx = \frac{\pi}{2}x + c$

96. (c) Area = $2\int_0^1 ydx$

$= 2\int_0^1 (1 - |x|)\,dx$

$= 2\int_0^1 (1 - x)\,dx = 2\left[x - \frac{x^2}{2}\right]_0^1$

$= 2\left(1 - \frac{1}{2}\right) - 0$

$= 1$

97.c 98.c 99.a 100.a

…The End…