Section - I

1. (a) Total no. of elements = 5.
No. of subsets having not more than 3 elements
$= c(5, 3) + c(5, 2) + c(5, 1) + c(5, 0)$
$= 10 + 10 + 5 + 1 = 16$

2. (d) $2\tan^2x = \sec^2x$
i.e. $2\tan^2x = 1 + \tan^2x$
i.e. $\tan^2x = 1$
i.e. $\tan^2x = \tan^2\frac{\pi}{4}$
$\therefore \quad x\pi \pm \frac{\pi}{4}$

3. (d) We have $\frac{a}{\sin A} = \frac{b}{\sin B}$
i.e. $\frac{3}{\frac{3}{4}} = \frac{4}{\sin B}$
i.e. $\sin B = 1$
$\therefore \quad B = 90°$

4. (c) $\vec{a} = \lambda\hat{a}$ i.e. $|\vec{a}| = \lambda|\hat{a}|$
i.e. $\lambda = |\vec{a}|$

5. (b) $1 - \log_e2 + \frac{(\log_e2)^2}{2} - \frac{(\log_e2)^3}{3!} + \ldots = e^{-\log_e 2}$
$= e^{\log_e \frac{1}{2}} = \frac{1}{2}$

6. (d) Sum of roots = 0
i.e. $-\frac{\lambda - 5}{\lambda - 2} = 0$
i.e. $\lambda = 5$

7. (b) We have $|A.\ \text{Adj}\ A| = |A|\ |I| = 8.1 = 8$

8. (b) Here $D = \begin{vmatrix} 1 & 3 \\ 4 & -1 \end{vmatrix} = -1 - 12 = -13 \neq 0$
So it has unique solution.

9. (c) The required line is $5(x - 2) - 3(y - 3) = 0$
i.e. $5x - 3y - 19 = 0$

10. (d) y-coordinates of centroid = 0
i.e. $\frac{a + b - 3}{3} = 0$
i.e. $a + b = 3$

11. (b) By definition 'b'

12. (a) Here $a^2 = 25$, $b^2 = 9$, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}}$
$= \sqrt{\frac{16}{25}} = \frac{4}{5}$

13. (d) Here $16x^2 - 9y^2 = 144$
i.e. $\frac{x^2}{9} - \frac{y^2}{16} = 1$
So $|PS_1 - PS_2| = 2.3 = 6$

14. (d) By definition, option 'd' is correct.

15. (b) $\lim\limits_{x \to 0} \frac{\sin ax - \sin bx}{x} = \lim\limits_{x \to 0} \left[\frac{\sin ax}{ax} . a - \frac{\sin bx}{bx} . b\right]$
$= 1.a - 1.b = a - b$

16. (d) $\lim\limits_{x \to 0} f(x) = f(0)$
i.e. $\cos 0 + 1 = K$ i.e. $K = 2$

17. (a) $e^{3\log_e x} = x^3$ and its derivative is $3x^2$

18. (c) $f'(x) = 2x - 4$
So, $f'(x) > 0$
i.e. $2x - 4 > 0$ i.e. $x > 2$

19. (b) Put $1 + e^x = y$ i.e. $dy = e^x dx$
Then $\int \frac{e^x dx}{1 + e^x} = \int \frac{dy}{dx} = \ln y + c = \ln(1 + e^x) + c$

20. (b) Req. area $= \int_1^2 dx = \int_1^2 x^3 dx = \left[\frac{x^4}{4}\right]_1^2 = \frac{16}{4} - \frac{1}{4}$
$= \frac{15}{4}$ sq. units

21. (b)

22. (a)

23. (b)
$$\begin{matrix} COOH \\ | \\ COOH \end{matrix} + H_2SO_4 \longrightarrow CO + CO_2 + H_2SO_4.H_2O$$

24. (b)

25. (a)

26. (b)

27. (d)

28. (b)

29. (c)

30. (d)

31. (a) $1NH_2O = 1MH_2O$
pH of $1MH_2O = 7$

32. (b) $W = \vec{F}.\vec{S} = (3\hat{i} + 4\hat{j} + 0\hat{k}).(0\hat{i} + 2\hat{j} - 3\hat{k})$
$= 0 + 8 - 0$
$= 8$ J

33. (b) $\frac{E_i}{E_f} = \frac{T \times n \times 4\pi r^2}{T \times 4\pi R^2} = n\left(\frac{r}{R}\right)^2$ ... (i)
Since volume remain same
So $n \times \frac{4\pi r^3}{3} = \frac{4\pi R^3}{3}$
or, $\frac{r}{R} = \left(\frac{1}{n}\right)^{\frac{1}{3}}$ .... (ii)
From (i) & (ii)
$\frac{E_i}{E_f} = n\left(\frac{1}{n}\right)^{\frac{2}{3}} = 1000\left(\frac{1}{1000}\right)^{\frac{2}{3}}$
$= 1000 \times \frac{1}{100} = 10:1$
$\therefore \quad \frac{E_f}{F_i} = \frac{1}{10}$

34. (b) wt = upthrust

or, $V\rho g = 70\%$ of $V\sigma g$

or, $\rho = 0.7$ g/cc

$\therefore$ Relative density = 0.7

35. (a) $F = mg\sin\theta + ma$

$= 200 \times \sin 30 + 20 \times 2$

$= 140$ N

36. (a) In metal energy transfer by conduction.

37. (a) Mechanical waves on the surface of liquid are transverse in nature.

38. (a)

39. (b) $\Delta L = 10\log\dfrac{1.26\ I}{I} = 1$ db

40. (b) $E \propto \dfrac{1}{r^2}$

or, $\dfrac{\Delta E}{E} = -2\dfrac{\Delta r}{r}$

$= -2 \times 2\%$

$= -4\%$

= 4% decreases

41. (a) $Bqv = \dfrac{mv^2}{r}$

or, $Bqr = mv$

or, $Bqr = \sqrt{2mE}$

or, $r \propto \sqrt{m}$

$m_p > m_e$ so $r_p > r_e$ i.e. path of proton is less curved.

42. (c) Specific resistance of conductor varies with temperature by $\rho_\theta = \rho_0\,(1 + \alpha\Delta\theta)$

43. (d) Electronic transition can't emit γ-rays.

44. (d) Penetrating power of x-rays depends on energy i.e. frequency.

45. (b) D_2 is reverse biased so doesnot conduct

$I = \dfrac{E}{20 + 30} = \dfrac{5}{50} = \dfrac{1}{10}$ A

46. (d) $V = \dfrac{Q}{4\pi\varepsilon_0 r} - \dfrac{Q}{4\pi\varepsilon_0 3r}$

$= \dfrac{Q}{4\pi\varepsilon_0 r} \times \dfrac{2}{3}$

or, $\dfrac{Q}{4\pi\varepsilon_0 r} = \dfrac{3V}{2}$ .... (i)

$E = \dfrac{Q}{4\pi\varepsilon_0 (3r)^2} = \dfrac{Q}{4\pi\varepsilon_0 r} \times \dfrac{1}{9r}$

$= \dfrac{3V}{2} \times \dfrac{1}{9r} = \dfrac{V}{6r}$

47. (c) For series

$10 = \dfrac{2R}{5 + 2r}$

or, $10(5 + 2r) = 2E$ ... (i)

2nd case

$8 = \dfrac{E}{5 + \frac{r}{2}}$

or, $8\left(5 + \dfrac{r}{2}\right) = E$ ... (ii)

From (i) & (ii)

$10(5 + 2r) = 2 \times 8\left(5 + \dfrac{r}{2}\right)$

or, $25 + 10r = 40 + 4r$

or, $6r = 15$

or, $r = \dfrac{15}{6} = 2.5\Omega$

48. (d) $E = \left|-\dfrac{d\phi}{dt}\right| = \dfrac{d(3t^2 + 4t + 2)}{dt} = 6t + 4$

$= 6 \times 2 + 4 = 16V$

49. (b) 50. (a) 51. (a) 52. (d) 53. (a) 54. (b)
55. (a) 56. (b) 57. (a) 58. (b) 59. (c) 60. (a)

Section - II

61. (a) $f(x) = \dfrac{\cos^2x + \sin^4x}{\sin^2x + \cos^4x} = \dfrac{\cos^2x + \sin^2x\,(1 - \cos^2x)}{\sin^2x + \cos^2x(1 - \sin^2x)}$

$= \dfrac{\cos^2x + \sin^2x - \sin^2x\cos^2x}{\sin^2x + \cos^2x - \sin^2x\cos^2x}$

$= \dfrac{1 - \sin^2x\cos^2x}{1 - \sin^2x\cos x} = 1$

62. (c) Here $\alpha + \beta = \dfrac{5}{6}$, $\alpha\beta = \dfrac{1}{6}$

So $\tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}\dfrac{\alpha + \beta}{1 - \alpha\beta} = \tan^{-1}\dfrac{\frac{5}{6}}{1 - \frac{5}{6}}$

$= \tan^{-1} 1 = \dfrac{\pi}{4}$

63. (a) Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|} = \dfrac{2.5 + 1.-3 + 2.1}{\sqrt{2^2 + 1^2 + 2^2}}$

$= \dfrac{10 - 3 + 2}{\sqrt{9}} = \dfrac{9}{3} = 3$

64. (a) No. of arrangements of CALCUTTA $= \dfrac{8!}{2!\,2!\,2!}$

$= 5040$

No. of arrangements of AMERICA $= \dfrac{7!}{2!} = 2520$

So ratio of arrangements = 5040 : 2520 = 2:1

65. (c) We have, $(1 + x)^n = c_0 + c_1x + c_2x^2 + c_3x^3 + ... + c_nx^n$

Putting x = 2, we get $(1 + 2)^n = c_0 + 2.c_1 + 2^2.c_2 + 2^3.c_3 + ... + 2^nc_n$

i.e. $c_0 + 2c_1 + 2^2.c_2 + 2^3.c_3 + ... + 2^n.c_n = 3^n$

66. (d) $\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\ldots$

$=\frac{1}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{15}\right)+\ldots\right]$

$=\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$

67. (b) Given $\left(\frac{1+i}{1-i}\right)^m=1$

i.e. $\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^m=1$

i.e. $\left(\frac{1+2i-1}{1+1}\right)^m=1$

i.e. $i^m=1\Rightarrow m=4$

68. (b) The condition for one slope equals n times the other is

$4nh^2=ab(1+n)^2$

Put x = 5, $4.5.h^2=ab.(1+5)^2$

i.e. $20h^2=36\,ab$

i.e. $5h^2=9ab$

69. (a) Here vertex = (a, 0)

i.e. h = a, K = 0

Focus = (a', 0) (b + α, K) = (a', 0)

i.e. $\alpha=a'-a$

So the equation of parabola is

$(y-K)^2=4\alpha(x-h)$

i.e. $y^2=4(a'-a)(x-a)$

70. (b) The equation of plane passing through the intersection of two planes is

$2x+3y+10z-8+\lambda(2x-3y+7z-2)=0$

which is perpendicular to $3x-2y+4z=5$, so $(2+2\lambda).3+(3-3\lambda).{-2}+(10+7\lambda).4=0$

i.e. $6+6\lambda-6+6\lambda+40+28\lambda=0$

i.e. $\lambda=-1$

So the required equation is $2x+3y+10z-8-2x+3y-7z+2=0$

i.e. $6y+3z-6=0$

i.e. $2x+z=2$

71. (b) For continuity, $\lim_{x\to0}\frac{1-\cos x}{x^2}=K$

i.e. $\lim_{x\to0}\frac{\sin x}{2x}=K\Rightarrow K=\frac{1}{2}$

72. (d) Here, $y=x^{x^x}$ i.e. $y=x^y$

$\ln y=y\ln x$ i.e. $\ln y-y\ln x=0$

$\therefore\ \frac{dy}{dx}=-\frac{-\frac{y}{x}}{\frac{1}{y}-\ln x}=\frac{y^2}{x(1-y\ln x)}=\frac{y^2}{x(1-\ln y)}$

73. (d) Here, a = 4, b = 9, $f(x)=\sqrt{x}$, $f'(x)=\frac{1}{2\sqrt{x}}$

So, $f(b)-f(a)=(b-a)f'(c)\Rightarrow 3-2=(9-4)\frac{1}{2}$

$\sqrt{c}\Rightarrow\sqrt{c}=2.5\ \therefore\ c=6.25$

74. (c) Here $I=\int\frac{x^5\,dx}{\sqrt{1+x^3}}=\int\frac{x^3.x^2\,dx}{\sqrt{1+x^3}}$

Put $y=14x^3$

$\therefore\ \frac{du}{3}=x^2dx$

Then $I=\int\frac{(y-1)\frac{du}{3}}{\sqrt{y}}=\frac{1}{3}\int\left[\sqrt{y}-\frac{1}{\sqrt{y}}\right]dy$

$=\frac{1}{3}\left[\frac{y^{3/2}}{\frac{3}{2}}-\frac{y^{1/2}}{\frac{1}{2}}\right]$

$=\frac{2}{9}(1+x^3)^{3/2}-\frac{2}{3}(1+x^3)^{1/2}+c$

75. (a) Given curve is $y=a\sqrt{x}+bx$.

It passes through the point (1, 2), so a + b = 2 ... (i)

Clearly the curve passes through origin. Also, area between the curve, line x = 4 and x-axis = 8 sq. units.

So $\int_0^4(a\sqrt{x}+bx)\,dx=8$

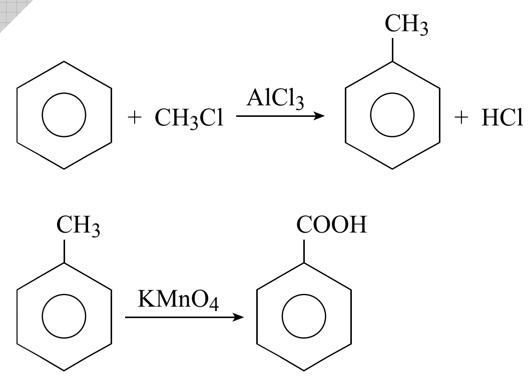
$\therefore\ \left.\frac{2}{3}ax^{3/2}+\frac{b}{2}x^2\right]_0^4=8$

$\frac{2}{3}a.8+\frac{b}{2}.16=8$

i.e. 2a + 3b = 3 .... (ii)

Solving (i) and (ii) we get a = 3, b = –1

76. (b)



77. (d) $Na_2S_2O_3+I_2\longrightarrow NaI+Na_2S_4O_6$
(aq) (aq)

78. (a)

$CaC_2+N_2\longrightarrow\underbrace{CaCN_2+C}_{\text{Nitrolim}}$

79. (b) $N_{mix} = \dfrac{10^{-2} + 10^{-4}}{2} = 5.05 \times 10^{-3}$ N

$= 5.05 \times 10^{-3}$ M

$pH = -\log[5.05 \times 10^{-3}] = 2.29$

80. (d) $[S^{--}] = 3S$

$S = \dfrac{[S^{--}]}{3} = \dfrac{6.2 \times 10^{-7}}{3} = 2.06 \times 10^{-7}$

$As_2S_3 = 108\ S^5 = 108 \times (2.06 \times 10^{-7})^5$

$= 4 \times 10^{32}$

81. (b) Mass of water = 90g

$\therefore$ EW of water during decomposition is 9

Hence, gram eq. of water $= \dfrac{90}{9} = 10$

= 10F is required.

82. (a) $\dfrac{x + 8}{x + 35.5} = \dfrac{3}{5}$

$x = \dfrac{3 \times 35.5 - 8 \times 5}{5 - 3} = 33.25$

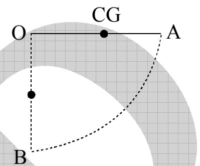
83. (b) Impulse = mu + mv

$= m(\sqrt{2gh_2} + \sqrt{2gh_1})$

$= 0.2\ (\sqrt{2 \times 10 \times 4} + \sqrt{2 \times 10 \times 5})$

$= 3.78$ kgm/s

84. (a)



$\Delta PE = \Delta KE$

or, $mg\dfrac{l}{2} = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

or, $mg\dfrac{l}{2} = \dfrac{1}{2}mv^2 + \dfrac{1}{2} \times \dfrac{1}{12}ml^2.\left(\dfrac{v}{\frac{l}{2}}\right)^2$

or, $\dfrac{mgl}{2} = \dfrac{1}{2}mv^2 + \dfrac{mv^2}{6}$

or, $\dfrac{mgl}{2} = \dfrac{3mv^2 + mv^2}{6}$

or, $mv^2 = \dfrac{3mgl}{4}$

or, $v = \dfrac{\sqrt{3gl}}{2}$

$\therefore$ Vel. of end $= 2v = 2\dfrac{\sqrt{3gl}}{2} = \sqrt{3gl}$

85. (c) $\dfrac{1}{2}mv^2 - \dfrac{GMm}{R} = -\dfrac{GMm}{R + h}$

or, $\dfrac{1}{2}m\,gR = GMm\left[\dfrac{1}{R} - \dfrac{1}{R + h}\right]$

or, $\dfrac{gR}{2} = gR^2\dfrac{(R + h - R)}{R(R + h)}$

or, $R + h = 2h$

or, $h = R$

86. (c) $\dfrac{\lambda}{2} = 0.2$

or, $\lambda = 0.4$ m

$n = \dfrac{l}{\lambda} = \dfrac{2}{0.4} = 5$

$f = n \times \dfrac{1}{2l}\sqrt{\dfrac{Tl}{M}} = 5 \times \dfrac{1}{2 \times 2}\sqrt{\dfrac{20 \times 2}{10^{-3}}} = 250$ Hz

87. (a) $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1.f_2}$

If power is 0 then

or, $\dfrac{1}{f_1} + \dfrac{1}{f_2} = \dfrac{d}{f_1.f_2}$

or, $\dfrac{f_1 + f_2}{f_1 f_2} = \dfrac{d}{f_1.f_2}$

or, $d = f_1 + f_2$

88. (b) $m = \dfrac{L}{f_0} \times \dfrac{D}{f_e} = \dfrac{10}{5} \times \dfrac{25}{0.5} = 100$

89. (b) $E = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{\varepsilon_0 A}{d}V^2$

$= \dfrac{1}{2} \times \dfrac{8.85 \times 10^{-12} \times 50 \times 10^6}{10^3} \times (100 \times 10^3)^2$

$= 2.2 \times 10^3$ J

90. (a) $\dfrac{R_2}{R_1} = \left(\dfrac{D_1}{D_2}\right)^4 = \left(\dfrac{D_1}{\frac{D_1}{3}}\right)^4 = 81 : 1$

91. (a) $B = \dfrac{\mu_0}{4\pi}\dfrac{M}{(d^2 + l^2)^{3/2}}$

or, $M = \dfrac{4 \times 10^{-6} \times 4\pi}{4\pi \times 10^{-7}} \times (0.03^2 + 0.4^2)^{3/2}$

$= 5 \times 10^{-3}\ Am^2$

$\therefore$ $M = 2ml$

or, $m = \dfrac{5 \times 10^{-3}}{2l} = \dfrac{5 \times 10^{-3}}{0.08} = 0.0625$ Am

92. (a) **1st case**

$\dfrac{1}{\lambda_B} = R\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right]$

$\dfrac{1}{\lambda_B} = \dfrac{5R}{36}$

or, $\lambda_B = \dfrac{36}{5R}$ ... (i)

2nd case

$$\frac{1}{\lambda_L} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

or, $\frac{1}{\lambda_L} = R\left[\frac{4-1}{4}\right]$

or, $\lambda_L = \frac{4}{3R}$ .... (ii)

Diving

$$\frac{\lambda_B}{\lambda_L} = \frac{36}{5R} \times \frac{3R}{4} = \frac{27}{5}$$

93. (a) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

or, $10\% = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

or, $\ln 0.1 = \frac{t}{T_{1/2}} \times \ln 0.5$

or, $t = T_{1/2} \times \frac{\ln 0.1}{\ln 0.5}$

$= 12.3 \times \frac{\ln 0.1}{\ln 0.5}$

$= 40.9$ yrs.

94. (a) $P = \frac{n}{t}\frac{hc}{\lambda}$

$$\frac{n}{t} = \frac{12 \times 248 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 1.49 \times 10^{19}$$

95. (b) At STP

$$C_{rms} = \sqrt{\frac{3P_0}{\rho_0}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{1.251}} = 492.1 \text{ m/s}$$

Again

$$\frac{C'_{rms}}{C_{rms}} = \sqrt{\frac{T'}{T}}$$

or, $C_{rms} = 492.1\sqrt{\frac{400}{273}}$

$= 595.6$ m/s

96. (d) 50% of $\frac{1}{2}mv^2 = ms\Delta\theta + mL_f$

or, $\frac{v^2}{4} = 127\,(330 - 27) + 2.42 \times 10^4$

or, $v = \sqrt{4(127 \times 303 + 2.42 \times 10^4)} = 500$ m/s

97. (d) 98. (c) 99. (a) 100. (c)

...The End...