Section – I

1.(a) $ma = v\frac{dm}{dt}$

or, $a = \frac{v}{m}\frac{dm}{dt} = \frac{1000 \times 100}{1000} = 100\ m/s^2$

2.(a) $\vec{v}_{rc} = \vec{v}_r - \vec{v}_c$

$= \vec{v}_r + (-\vec{v}_c)$

3.(d) $\frac{2}{5}mR^2 = mK^2$

$\therefore\ K = \sqrt{\frac{2}{5}}R$

4.(c) Loss in wt = upthrust

or, $w - \frac{w}{2} = v\sigma_w$

or, $v = \frac{w}{2}$

$\therefore$ Density $= \frac{w}{v} = \frac{w}{w} \times 2 = 2$ g/cc

5.(c) $\frac{1.004P}{P} = \frac{T+1}{T}$

or, $1.004\ T = T + 1$

or, $T = \frac{1}{0.004} = 250$ K

6.(d) $\frac{\Delta l}{l} = \alpha\Delta\theta$

or, $\Delta\theta = \frac{1}{100 \times 2 \times 10^{-5}}$

$= \frac{10^3}{2} = 500°C$

7.(a)

8.(a) $v = \frac{fu}{u-f}$ & $m = \frac{f}{u-f}$

u is –ve & u < f so v is +ve & m > 1

9.(a) $L = 10\log\frac{I}{I_0} = 10\log\frac{5 \times 10^{-7}}{10^{-12}} = 56.9$ db

10.(b)

11.(a) $c = \frac{\varepsilon A}{d}$, c decreases if d increases

12.(c) $V = \frac{Q}{4\pi\varepsilon_0 r}$, $V \propto \frac{1}{r}$

13.(b) $\frac{2E}{R+2r} = \frac{E}{R+\frac{r}{2}}$

or, $\frac{2}{3+2r} = \frac{1}{3+\frac{r}{2}}$

or, $\frac{2}{3+2r} = \frac{2}{6+r}$

or, $12 + 2r = 6 + 4r$

or, $2r = 6$

or, $r = 3\Omega$

14.(d) In magnetic field no force act tangle is 0° & in electric field force act opposite to field i.e. retard.

15.(b) $M = m \times 2L$

or, $m = \frac{M}{2L}$

After bending

$M' = m \times AB = \frac{M}{2L} \times L = \frac{M}{2}$

16.(b) $2d\sin45° = 2\lambda$

or, $d = \frac{\lambda}{\sin45°} = \sqrt{2}\,\lambda$

17.(b) $\beta = 40 = \frac{I_c}{I_b}$

or, $40 = \frac{I_e - I_b}{I_b}$

or, $40\ I_b = I_e - I_b$

or, $I_b = \frac{I_e}{41} = \frac{8.2}{41} = 0.2$ mA

18.(a)
19.(b)
20.(a)
21.(c)
22.(a)
23.(b)
24.(d)
25.(b)
26.(b)
27.(b)
28.(b)
29.(b)
30.(c)
31.(b)
32.(a)
33.(c)

34.(d) $\int \cos y\ dy = \sin y + c = x + c$ if $y = \sin^{-1}x$

35.(d) $\vec{a} \times \vec{b}$ is vector perpendicular to both a and $\vec{b}$.

36.(b) L' Hospital Rule $= \lim_{x \to 0} \frac{ae^{ax} + be^{-bx}}{\cos x} = a + b$

37.(c) $y = 2\log_e x \Rightarrow \frac{dy}{dx} = \frac{2}{x}$

38.(b) e^x only obeys the statement.

39.(a)
40.(b)
41.(a)
42.(d)
43.(a)

44.(c) $\vec{a} + \vec{b} = \vec{c}$

$(\vec{a} + \vec{b})^2 = -\vec{c}^2$

$\therefore \quad \vec{a}^2 + 2\vec{a}.\vec{b} + \vec{b}^2 = \vec{c}^2$

i.e. $9 + 2 \times 3 \times 5 \cos\theta + 25 = 49$

or, $30 \cos\theta = 15$

$\theta = \frac{\pi}{3}$

45.(d) It is true whereas the others are not

46.(a) It makes 0 = 0, and identity.

47.(a)

48.(d)

49.(d)	50.(c)	51.(a)	52.(c)	53.(d)	54.(a)
55.(b)	56.(b)	57.(a)	58.(d)	59.(b)	60.(a)

Section – II

61.(b) For vertical motion

$y = u_y t - \frac{1}{2} g t^2$

or, $20.4 = u_y \times 2.5 - \frac{1}{2} \times 10(2.5)^2$

or, $u_y = 20.7$ m/s

Here $u = \sqrt{u_x^2 + u_y^2}$

or, $u_x = \sqrt{u^2 - u_y^2} = \sqrt{40^2 - 20.7^2} = 34.2$ m/s

$\therefore \quad R = \frac{2u_x u_y}{g} = \frac{2 \times 34.2 \times 20.7}{10}$

$= 141.5$ m

62.(b) $a = \frac{0^2 - 100^2}{2 \times 4} = \frac{0^2 - u^2}{2 \times 9}$

or, $\frac{100^2}{4} = \frac{u^2}{9}$

or, $u = \sqrt{\frac{9}{4} \times 100^2} = 150$ m/s

63.(d) Loss in wt = upthrust

$(80 + 12.4) - 20.5 = (V_m + V_w)\sigma_w$

or, $71.9 = V + \frac{12.4}{0.9}$ or, $V = 58.1$ cc

$\therefore \quad \rho = \frac{m}{V} = \frac{80}{58.1} = 1.37$ g/cc

64.(c) $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

or, $\frac{(P_a + P_w) \times 1}{285} = \frac{P_aV_2}{308}$

or, $V_2 = \left(\frac{(1.01 \times 10^5 + 10^3 \times 10 \times 40)\ 308}{285 \times 1.01 \times 10^5}\right)$

$= 5.3$ cc

65.(c) **1st case**

$\eta_1 = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$

or, $\frac{40}{100} = 1 - \frac{T_2}{800}$

or, $\frac{T_2}{800} = 1 - \frac{2}{5}$

or, $T_2 = \frac{3}{5} \times 800 = 480$ K

2nd case

$\eta_2 = \left(1 - \frac{T_2'}{T_1}\right) \times 100\%$

or, $\frac{50}{100} = 1 - \frac{T_2'}{800}$

or, $\frac{T_2'}{800} = \frac{1}{2}$

or, $T_2' = 400$K

$\therefore \quad \Delta T = T_2 - T_2' = 480\text{ K} - 400\text{ K} = 80$ K

66.(d) **For glass**

$R_1 = 5$ cm $\quad R_2 = -10$cm

$\frac{1}{fg} = (\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$= (1.5 - 1)\left(\frac{1}{5} - \frac{1}{10}\right)$

$= \frac{0.5}{10} = \frac{1}{20}$

or, $fg = 20$ cm

For water

$R_1 = \infty$, $R_2 = 10$cm

$\frac{1}{f_w} = (\mu_w - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$= \left(\frac{4}{3} - 1\right)\frac{1}{10} = \frac{1}{30}$

$\therefore \quad f_w = 30$ cm

$\therefore \quad \frac{1}{F} = \frac{1}{f_g} + \frac{1}{f_w} = \frac{1}{20} + \frac{1}{30}$

or, $F = \frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12$ cm

67.(c) $3.5\beta = 3.5 \times 10^{-3}$

or, $\frac{D\lambda}{d} = 10^{-3}$

or, $\lambda = \frac{10^{-3} \times 10^{-3}}{2.5}$

$= 4 \times 10^{-7}$ m

$= 400 \times 10^{-9}$ m

$= 400$ nm

68.(b) $PE = 9 \times 10^9 \frac{Q_1Q_2}{r}$

$= \frac{9 \times 10^9 \times 2 \times 10^{-5} \times 2 \times 10^{-5}}{0.1}$

$= 36$ J

69.(d) $R_{50} = 5\ \Omega$, $R_{100} = 6\ \Omega$

$$\alpha = \frac{R_{100} - R_{50}}{R_{50}(100 - 50)}$$
$$= \frac{6-5}{5 \times 50} = 4 \times 10^{-3}/°C$$

Again $R_0 = R_{50}\{1 + (0 - 50)\alpha\}$
$$= 5(1 - 5 \times 4 \times 10^{-3})$$
$$= 4\Omega$$

70.(a) $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$

When direction is reversed then direction of force also reversed as

$$F' = -\frac{\mu_0}{2\pi}\frac{2I_1I_2}{3d} = -\frac{2F}{3}$$

71.(c) $\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R}$

or, $\phi = \tan^{-1}\left(\frac{2\pi fL}{R}\right)$
$$= \tan^{-1}\left(\frac{2\pi \times 50 \times 0.21}{12}\right) = 80°$$

72.(d) **1st case**

$$\frac{hc}{\lambda} = \phi + \frac{1}{2}mv^2$$

or, $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$ ….(1)

2nd case

$$4\frac{hc}{3\lambda} = \phi + \frac{1}{2}mv'^2$$

or, $\frac{1}{2}mv'^2 = \frac{4hc}{3\lambda} - \phi$ … (2)

Diving, $\frac{v'}{v} = \sqrt{\frac{\frac{4hc}{3\lambda} - \phi}{\frac{hc}{\lambda} - \phi}}$

or, $v' = v = \sqrt{\frac{\frac{4hc}{3\lambda} - \phi}{\frac{hc}{\lambda} - d}}$

Check taking numerical value of $\frac{hc}{\lambda}$ & ϕ

73.(b) When $\left(\frac{2}{3}\right)$ of sample decayed then

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t_1/T_{1/2}}$$

or, $\frac{1}{3} = \left(\frac{1}{2}\right)^{t_1/T_{1/2}}$

or, $\frac{t_1}{T_{1/2}} = \frac{ln\left(\frac{1}{3}\right)}{ln\left(\frac{1}{2}\right)}$

$t_1 = 31.7$ min

2nd case when $\left(\frac{1}{3}\right)$ of sample decayed then

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t_2/T_{1/2}}$$

or, $\frac{2}{3} = \left(\frac{1}{2}\right)^{t_2/T_{1/2}}$

or, $t_2 = 11.7$ min

$\Delta t = t_1 - t_2 = 31.7 - 11.7$
$= 20$ min

74.(b) $2f_0^s = f_0$

or, $2 \times \frac{1}{2l_s}\sqrt{\frac{Tl}{M}} = \frac{v}{4l_p}$

or, $\frac{1}{0.5}\sqrt{\frac{50 \times 0.5}{M}} = \frac{320}{4 \times 0.8}$

or, $\frac{25}{M} = 50^2$

or, $M = \frac{25}{50^2} = 0.01$ kg
$= 10g$

75.(b)
76.(a)
77.(a)
78.(c)
79.(d)
80.(a)
81.(c)
82.(c)

83.(a) Since $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$

$\therefore\ \sin\theta + \cos\theta = \frac{b}{c}$

and $\sin\theta\cos\theta = \frac{c}{a}$

Now, $(\sin\theta + \cos\theta)^2 = \frac{b^2}{a^2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$

$1 + 2\frac{c}{a} = \frac{b^2}{a^2}, \frac{a + 2c}{a} = \frac{b^2}{a^2}$

$a + 2c = \frac{b^2}{a}, a^2 + 2ca = b^2$

$a^2 - b^2 = -2ca$

$b^2 - a^2 = 2ac$

84.(b) Given, $x + \frac{1}{x} = 2$

$X^2 + 1 = 2x$

This is only valid when x = 1

i.e. $\sin^{-1}(1) = \frac{\pi}{2}$

85.(d)

86.(c) If $f(x) = \int e^x(x-1)(x-2)\,dx$, then

$f'(x) = e^x (x-1)(x-2)$

e^x is always positive

So, $f'(x) > 0$ for $x - 1 > 0$ and $x - 2 > 0$

So, f(x) is increasing on R – [1, 2]

87.(b) The equation of the circle passing through (0,0), (1,0), (0,1) is

$x^2 + y^2 - x - y = 0$

So, $k^2 + 4k^2 - k - 2k = 0$

$5k^2 - 3k = 0$

$k = 0, \frac{3}{5}$

88.(d) $\log_e\frac{1-x^3}{1-x} = \log_e(1-x^3) - \log_e(1-x)$

Coeff. of x^9 = coeff. of $(x^3)^3$ in the first – coeff of x^9 in the second

$= -\frac{1}{3} - \left(-\frac{1}{9}\right) = -\frac{1}{3} + \frac{1}{9} = -\frac{2}{9}$

89.(b) $T = S_1$

i.e. $x \times (-1) + y \times (2) - 9 = (-1)^2 + 2^2 - 9$

or, $-x + 2y - 5 = 0$

or, $x - 2y + 5 = 0$

90.(a) If the point of contact is (x_1, y_1), the equation of the tangent is

$yy_1 = 2a(x + x_1)$

or, $y = \frac{2a}{y_1}x + \frac{2ax_1}{y_1}$

This must be identical with

$y = mx + \frac{a}{m}$

or, $\frac{m}{\frac{2a}{y_1}} = \frac{\frac{a}{m}}{\frac{2ax_1}{y_1}}$

or, $\frac{my_1}{2a} = \frac{ay_1}{2amx_1}$

or, $x_1 = \frac{a}{m^2}$

91.(a) $2ae = 6 \Rightarrow ae = 3$. Also, $2b = 8 \Rightarrow b = 4$

So, $b^2 = 16$

or, $a^2(1 - e^2) = 16$

or, $a^2 - a^2e^2 = 16$

or, $a^2 = 16 + 9 = 25$

So, the ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

92.(a) Since f (x) is continuous at x = 2

$\therefore\ f(2) = \lim_{x \to 2} f(x)$

$2 = \lim_{x \to 2} \frac{x^2 - (a+2)x + a}{x - 2}$

$2 = \lim_{x \to 2} \frac{x(x-2) - a(x-1)}{x - 2}$

Which is true if a = 0

93.(c) We know, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$

$\therefore\ y = e^x$

$\frac{dy}{dx} = ex = y$

94.(b) $y = \frac{x^3}{8a^2}$

$\frac{dy}{dx} = \frac{3x^2}{8a^2}$

We have, Slope of normal $= -\frac{1}{\frac{dy}{dx}} = -\frac{8a^2}{3x^2}$

Given, Slope of normal $= -\frac{2}{3}$

$-\frac{8a^2}{3x^2} = -\frac{2}{3} \Rightarrow x^2 = 4a^2 \Rightarrow x = 2a$

$[\therefore\ a > 0\ \&\ x > 0]$

$\therefore\ y = a$

$\therefore$ Required point is (2a, a)

95.(a) $\int \frac{dx}{\sqrt{\frac{17}{4} - \left(x^2 - 2*x*\frac{3}{2} + \frac{3}{2}\right)^2}}$

$= \sin^{-1}\frac{x - \frac{3}{2}}{\frac{\sqrt{17}}{2}} + c$

$= \sin^{-1}\frac{2x-3}{\sqrt{17}} + c$

$= fog(x) + c$

96.(d)

97.(b) 98.(c) 99.(c) 100.(c)

…Best of Luck…